The Study of the Spatial Coherence of Surface Waves by the Nonlinear Green-Naghdi Model in Deep Water

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LONG-TERM GOAL

The goal is to identify the role of nonlinear wave interaction in the spatial coherence of ocean waves. For this purpose, a numerical tool based on the Irrotational Green-Naghdi (IGN) model to simulate short-crested sea-state is developed.

OBJECTIVES:

The objectives are to develop a higher-level Green-Naghdi model; to provide oceanographers and/or ocean engineers a new, numerical nonlinear-wave model; and to simulate fully-nonlinear interaction in short-crested random ocean surface efficiently. Emphasis will be made on the optimization between accuracy and computational effort by adjusting the `Level' of the model.

APPROACH

The irrotational version of the GN model (IGN model), as developed by Kim et al.(2000), will be used to model the nonlinear evolution of ocean waves. The accuracy of the model can be controlled by the 'Level′ of the model, which is defined as the number of interpolation functions in the vertical direction. As the Level increases, the model describes the physics more accurately with a penalty of higher computational effort. The Level is chosen such that simulations can be performed with minimal effort and redundancy in accuracy. The optimized model is discretized by a pseudo-spectral method on the horizontal plane.

WORK COMPLETED

The first year of the project was devoted to the derivation, numerical implementation, and the validation of the IGN model. The general derivation of the theory is submitted to the Journal of Engineering Mathematics and is in print. The theory is also applied to the hydroelastic problem of a mat-type structure and several publications have resolved from this work (also related to an NSF project). Regarding the numerical implementation, pseudo-spectral codes for two- and three-dimensional problems have been developed. Validation of the theory and the numerical code was made from the comparisons with known exact solutions and experiments. The agreement is good.

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In the second year, a numerical code for the three-dimensional problem is developed to simulate the random sea surface for a given directional spectrum. As output, the surface elevation and velocity field are provided. Postprocessors to evaluate the statistics of the wave data, such as the significant wave height, are also developed. Monaldo's technique (1999) to measure the crest length is being implemented. The developed three-dimensional code is applied to the simulation of the directional sea spectrum and swell.

RESULTS

Generation of rogue waves by Class I instability

Rogue waves can be generated from the Stokes wave train modulated by a side-band instability (see, e.g. Kim & Ertekin, 1999 and Osborne, 1999). Osborne's rogue wave solution of NLS equations shows that the maximum wave height of the rogue waves is 2.4 times the wave height of the carrier wave. Since NLS equations are derived under the assumption of weak nonlinearity, it is expected that the value 2.4 may not be a universal number. For the initial steepness of 0.06 to 0.17, the maximum growth of the wave height is investigated by IGN Level III equations. The length of the computational domain is taken as 10 times the carrier wave length. The Stokes wave is modulated by adding sinusoidal waves with wavelengths 90% and 110% of the carrier wave, and with the amplitudes 5 % of the carrier wave. Table 1 shows the results of the maximum wave height. When the steepness of the Stokes wave is small, the amplification due to the nonlinear modulation is about 2.4, which agrees well with Osborne's analytical result. As the steepness increases, the amplification increases to reach the maximum value 3.1 at $ka = 0.1$. Thereafter, the amplification decreases as the steepness increases.

Table 1. The maximum wave height due to the Class I instability in modulated Stokes' wave train

ka	$k\,H_{max}$	$H_{max}/2a$
0.06	0.29	2.4
0.07	0.36	2.6
0.08	0.45	2.8
0.09	0.53	2.9
0.10	0.62	3.1
0.11	0.66	3.0
0.12	0.65	2.7
0.13	0.65	2.5
0.14	0.68	2.4
0.17	0.75	2.2

The evolution of the wave elevation and its Fourier transformation, when $ka = 0.1$, is depicted in Fig. 1(a) and (b). The maximum value of the crest height and the minimum value of the trough elevation are also given in Fig. 1(c). The growth of the side-band instability at the early stage leads to a strong modulation at $t = 150T$. More strong modulation is observed at $t = 315T$. Thereafter, permanent down shifting in wave number and frequency is observed. When the strong modulation is observed in the physical space, growth of the unstable mode with the continuous spectrum is observed in the Fourier space. One may conjecture that this unstable mode is related to the unstable mode in Osborne's solution.
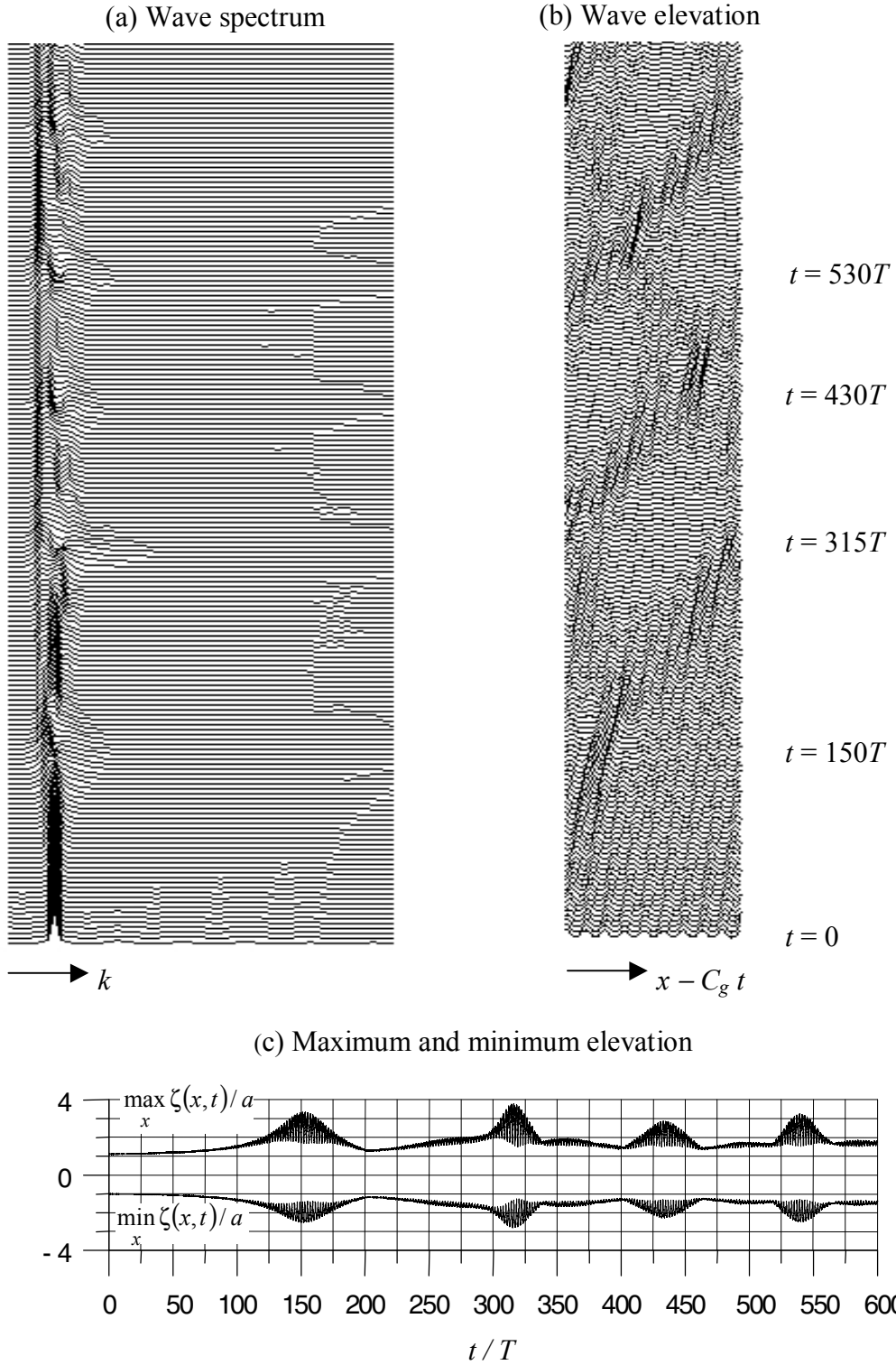
(a) Wave spectrum   (b) Wave elevation

$t = 530T$

$t = 430T$

$t = 315T$

$t = 150T$

$t = 0$

k $x - C_g\, t$

(c) Maximum and minimum elevation

$\max\limits_{x} \zeta(x,t)/a$

$\min\limits_{x} \zeta(x,t)/a$

$t\,/\,T$

Fig. 1 Self-modulation of the Stokes wave of ka = 0.1: (a) Spectrum amplitude in Fourier space (b) Wave elevation in physical space   (c) Spatial maximum and minimum of wave elevation

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Simulation of random seas with a directional spectrum

The nonlinear interaction in a short-crested random sea can be simulated for a given directional spectrum. The generation of rogue waves and increase of the crest length due to the nonlinear wave interaction are investigated. Fig.2 shows a typical result. The Bretschneider spectrum with the significant wave height of 14 m and the peak period of 14 s is used. Computations are made in a 3 km by 3 km domain. Extreme wave height up to 2.2 times the significant wave height was observed in most of the simulations. However, the extreme wave heights are usually observed at the short crests accompanied by Class II instability (see, e.g., inside the circle in Fig. 2(b)), which oppose the real sea observations that rogue waves have long crest lines. Presumably, we may need longer simulations to build the crest length. Another possible explanation is that the long-crestedness of the rogue wave is due to the swell component in the sea. The rogue waves are usually generated in storm seas, where the swell is common. In the next section, we will show that adding a swell component can generate long-crested rogue waves. Increase of the crest length due to the nonlinearity was also observed, as can be seen visually in the nonlinear result in Fig. 2(b). A quantitative measure of the crest length is made by the use of Monaldo's technique (Monaldo, 1998). In Fig. 3, the number of wave crests longer than the given crest lengths is plotted. The linear and nonlinear data for 6 different time stages, $t / T_p = 25$, 30, …,55, are plotted with the trend lines. The nonlinear results show longer crest lengths. However, the increase is minor than has been observed in Monaldo's comparison, where the field data are compared with the linear simulation results. As a matter of fact, in certain simulations with different initial conditions, the nonlinear results have not shown any increase of the crest lengths. More extensive simulation is necessary before the final conclusion is made on the effect of nonlinearity on the crest length[1].
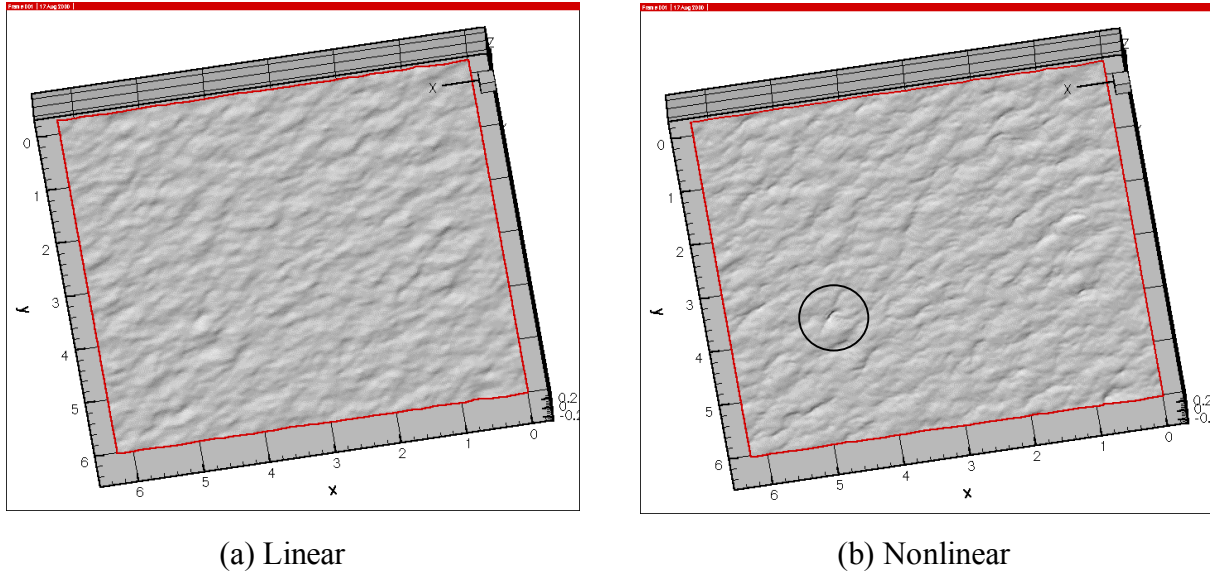

(a) Linear   (b) Nonlinear

Fig. 2 Perspective views of the surface elevation.

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[1] For example, the simulations can also be made by use of a spreading factor other than $\cos^2\theta$ that has been used in this report.
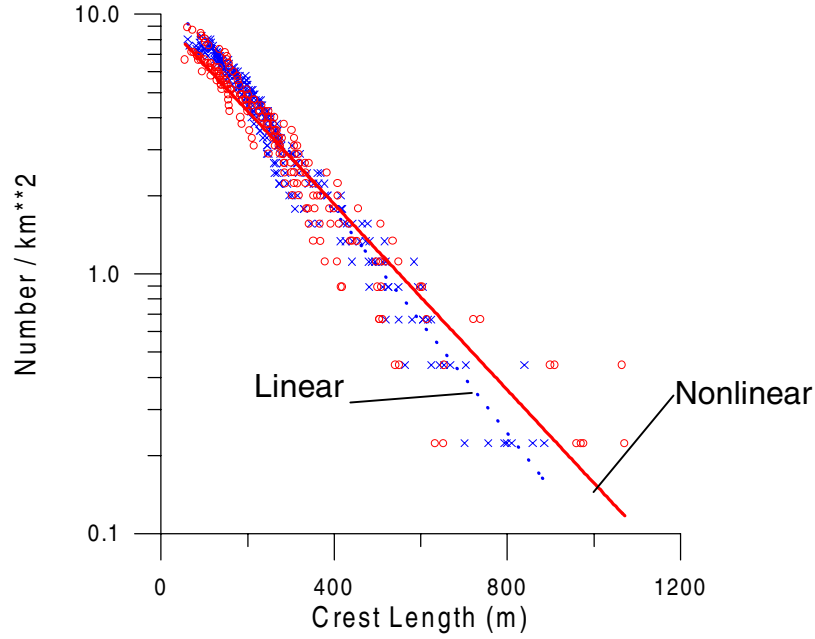
Fig. 3 The number of wave crests longer than the abscissa per square kilometer. Red circle and solid line are nonlinear result and trend lines, respectively. Blue x-marker and dotted line are from linear results.

Rogue wave in "The Perfect Storm"

A wave height of 100 ft was recorded in the North Atlantic during the storm caused by Hurricane Grace in October 1991 (see, e.g., "The Perfect Storm" by Sebastian Junger for details). We tried to simulate this freak wave. The initial condition for the storm sea is modeled by the linear sum of swell and short crested sea components. The swell component is modeled by a modulated Stokes wave of initial steepness $ka = 0.1$. The modulated wave at $t/T = 200$ (see Fig. 1) is taken as the initial condition. The period and wavelength of the swell is taken as 14 s and 300 m, respectively, which is taken from Hurricane Bonnie data (Walsh, 1998). Then the wave height of the swell component is about 10 m. Sea component with the Bretschneider spectrum and $\cos^2\theta$ spreading, with the significant wave height and peak period of 10 m and 14 sec, respectively, is added to the swell component. Simulations are made up to $t = 1800$ sec.
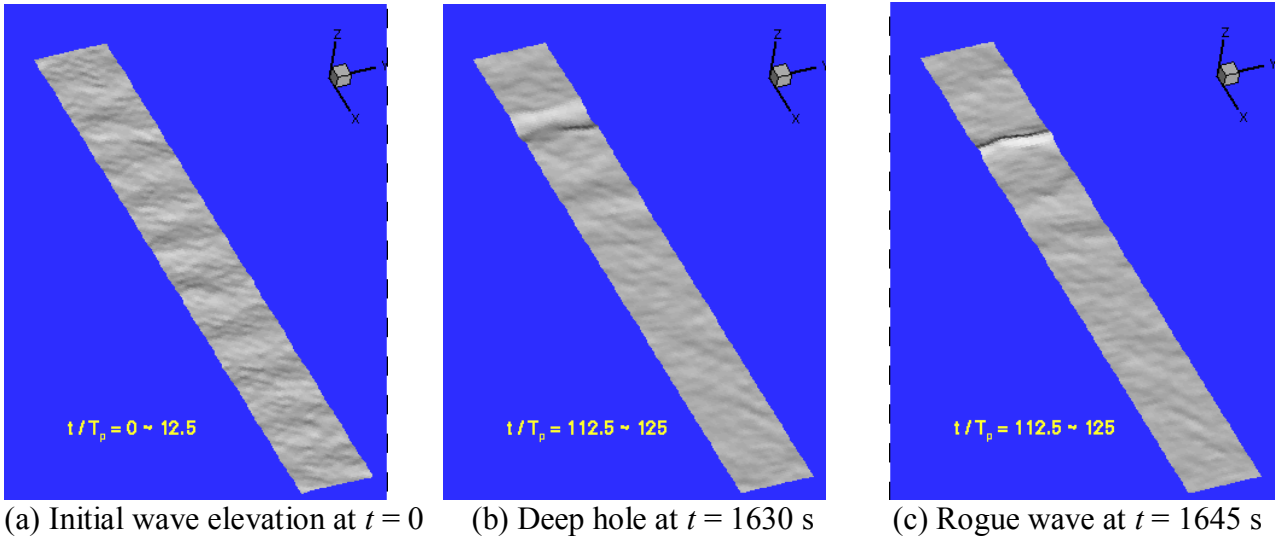
(a) Initial wave elevation at $t = 0$ (b) Deep hole at $t = 1630$ s   (c) Rogue wave at $t = 1645$ s

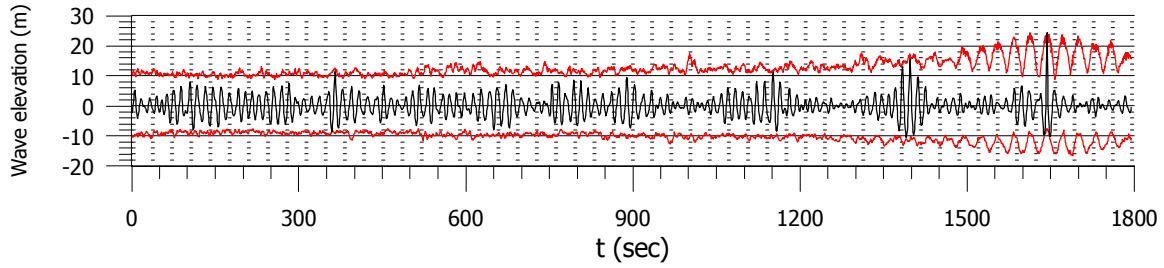*Fig. 4 Perspective views of the surface elevation.*

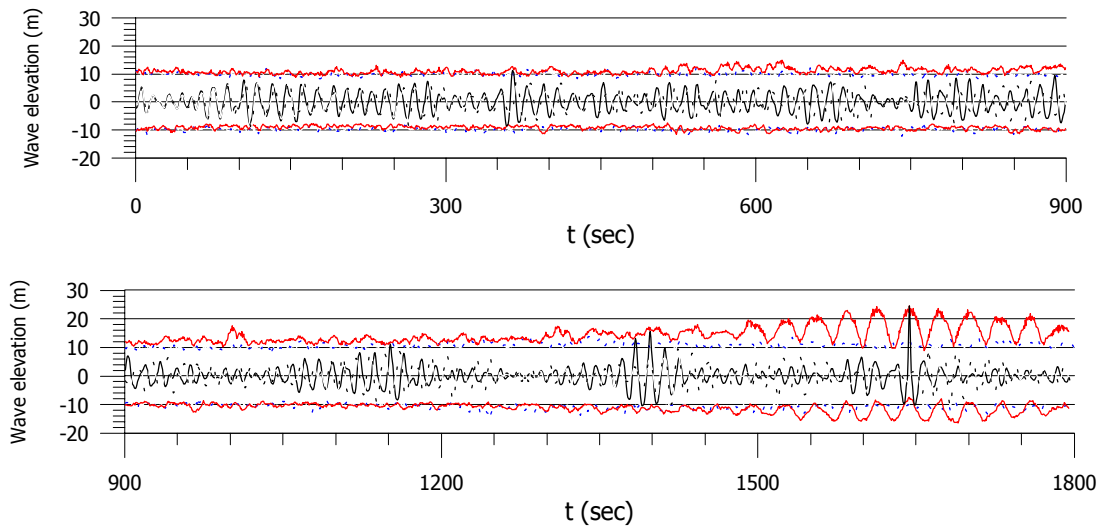*Fig. 5 Time record of the wave elevation at a point where maximum elevation occurred*

Fig. 6 Comparison of linear and nonlinear results
Wave elevation: —— *nonlinear,* ------ *linear*
Min. Max. elevation: —— *nonlinear,* ----- *linear*

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Fig. 4 shows the surface elevation at three different time steps. At around $t = 1600$ sec, the rogue wave of wave height 34 m is generated (Fig. 4(c)). Before the rogue wave reaches its peak, a deep hole is also observed as in the real sea observations (Fig. 4(b)). The time record of the wave elevation at a fixed location is shown in Fig. 5. Also shown are the maximum and the minimum values of the wave elevation at each time. At this location, the significant wave height is 14 m and maximum wave height is 34 m, which is more than 2.4 times the significant wave height. In Fig. 6, the same data are compared with the linear results. In the linear result, the significant wave height is 13 m and the maximum wave height is 20 m, which is only 1.5 times the significant wave height.

IMPACT/APPLICATION

A new numerical model for nonlinear evolution of ocean waves is developed. The new model can provide the wave environmental input for designing very large floating structures such as a Mobile Offshore Base (MOB).

TRANSITIONS

The shallow water version of the GN theory is applied to the hydroelastic problem of mat-type structures. The developed numerical codes, GNplate and EigPlate, was used in NSF project: Grant No.: BES-9532037, Co-PIs: H.R. Riggs and R.C. Ertekin, to evaluate the performance of floating runways.

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